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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, May 2014
(2008 Scheme)
Branch : ELECTRONICS AND COMMUNICATION
08.401 : Engineering Mathematics – III – Probability and Random Processes (TA)

Time: 3 Hours Max. Marks: 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. A discrete random variable X has the following probability distribution :

X :	0	1	2	3	4	5	6	7	8
F (x) :	a	3a	5a	7a	9a	11a	13a	15a	17a

   Find a, P (X < 3) and mean.

2. Find the mean and standard deviation of the normal distribution

   $f(x) = C\exp\left\{-\frac{1}{24}\left(x^2 - 6x + 4\right)\right\}$ where $-\infty < x < \infty$.

3. If X is uniformly distributed in $\left(-\frac{5}{4}, \frac{5}{4}\right)$, find $P\left[X < \frac{1}{2}\right]$.

4. A random sample of size 100 is taken from an infinite population having mean $\mu = 76$ and variance $\sigma^2 = 256$. What is the probability that $\overline{X}$ will lie between 75 and 78 ?

5. Suppose that X (t) is a Poisson process with E [X (9)] = 6. Find P [X (2) $\leq$ 3].

P.T.O.

6. Find the mean and variance of the stationary process with no periodic components and $R_x(\tau) = 25 + \frac{4}{1+6\tau^2}$.

7. If $X(t) = \sin(\omega t + \phi)$ where ϕ is uniformly distributed in $(0, 2\pi)$, show that X (t) is wide sense stationary.

8. If the auto-covariance function of a stationary process X (t) is given by $C(\tau) = A\, e^{-\alpha|\tau|}$, prove that X (t) is mean ergodic.

9. A stationary zero mean process X (t) has auto correlation function $R(\tau) = 10\, e^{-\tau^2/10}$. Find the mean and variance of $\mu_T = \frac{1}{5}\int_0^5 X(t)\, dt$.

10. Define the power spectrum of a wide sense stationary process X (t). Find the power spectrum of the WSS X (t) whose auto correlation function is $R(\tau) = e^{-\alpha|\tau|}$.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

MODULE – I

11. a) Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show a five or six.

b) The average test marks in a particular class is 79 and SD is 5. If the marks are normally distributed, how many students in a class of 200 did not receive marks between 75 and 82.

c) The joint density of X and Y is given by $f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \le x, y \le 1 \\ 0, & \text{otherwise} \end{cases}$

Find :

i) $P\left[0 < X < \frac{1}{2}, \frac{1}{4} < Y < \frac{1}{2}\right]$

ii) $P\left[Y > \frac{1}{2} \middle| X = \frac{1}{4}\right]$

iii) The conditional density f (y | x).

12. a) Fit a Poisson distribution to the following data :

x :	0	1	2	3	4
f (x) :	63	28	6	2	1

b) A fair die is tossed 180 times. Use Chebyshev's inequality to find a lower bound for the probability of getting 20 to 40 sixes.

c) Find the coefficient of correlation from the following data :

x :	218	220	236	225	220	227	228
y :	12.3	12.7	12.0	12.2	12.7	12.1	12.0

MODULE – II

13. a) Establish the necessary and sufficient conditions for the wide sense stationary of the process $X(t) = A \cos \omega t + B \sin \omega t$ where A and B are random variables.

b) If $X(t) = \sin(\omega t + \phi)$ where ϕ is uniformly distributed in $(0, 2\pi)$, show that X (t) is wide-sense stationary.

14. a) If X (t) and Y (t) are independent wide sense stationary process with zero mean, find the auto correlation function of

i) $Z(t) = a + b\,X(t) + c\,Y(t)$ and

ii) $Z(t) = a\,X(t) \cdot Y(t)$.

b) If $X(t) = A \cos t + B \sin t$ and $Y(t) = B \cos t + A \sin t$ where A and B are independent random variables with $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = 1$. Show that X (t) and Y (t) are not jointly wide sense stationary.

MODULE – III

15. a) The power spectrum of a zero mean wide sense stationary process is given by $S(w) = \begin{cases} 1 & \text{if } |w| < w_0 \\ 0 & \text{otherwise} \end{cases}$. Show that X (t) and $X\left(t + \frac{\pi}{w_0}\right)$ are uncorrelated.

b) The tpm of a Markov chain $\{X_n\}$ having 4 states 0, 1, 2 and 3 is

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0 & 0.2 & 0 & 0.8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ and the initial distribution is $P(X_0 = i) = \frac{1}{4}$ for

i = 0,1, 2, 3.

Find :

i) $P[X_2 = 2, X_1 = 1 | X_0 = 2]$

ii) $P[X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 2]$

iii) $P[X_2 = 3]$.

16. a) Find the average power of the random process if its spectral density is

$$S(w) = \frac{10\,w^2 + 35}{(w^2 + 4)(w^2 + 9)}.$$

b) What is the classification of Markov Process ?

c) Assume that a man is at an integral point of the x-axis between the origin and the point 4. He takes a unit step either to the right with probability 0.6 or to the left with probability 0.4 when he is at the origin, he takes a step to the right to reach 1 or he is at the point 4 when he takes a step to the left to reach 3. Find the transition probability matrix.
